2019

PHYSICS

(Honours)

Paper : PHY–HC–1016

(Mathematical Physics—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question Nos. **1**, **2** and **3** which are compulsory and *any* **three** from the rest

1. Fill in the blanks/Write True or False of the following : $1 \times 7 = 7$

(a) Length of a vector under rotation remains _____.

(b) Cross product of two vectors is always _____ to the plane containing the two vectors.

(c) The divergence of a vector depends only on the point (and the vector) in space, and not on the particular choice of the coordinate system.

(d) The integration of Dirac-delta function over whole space is _____.

(e) If a solution of homogenous linear differential equation is multiplied by a constant, the resultant function is also a solution of the equation.

(f) $\overline{\nabla} \cdot (f \overline{v}) =$ _____.

(g) If $f(x)$ is the probability distribution function, then its mean is defined as $\mu =$ _____.

2. Answer the following questions : 2×4=8

(a) If curl of a vector is zero, what does this signify physically? Illustrate with the help of an example.

(b) Under what physical situations, Dirac-delta function can be used? Explain with an example.

(c) Giving an example, explain the meaning of orthogonal curvilinear coordinate system.

(d) Differentiate between ordinary and partial differential equations giving an example of each.

3. Answer any *three* of the following : 5×3=15

(a) Find the unit vectors perpendicular to the plane

$$4x + 2y + 4z = -7$$

(b) Find the volume of a tetrahedron with $\vec{a}$, $\vec{b}$, $\vec{c}$ as adjacent edges (w.r.t. right-handed Cartesian coordinates), where

$$\vec{a} = \hat{i} + 2\hat{k}, \quad \vec{b} = 4\hat{i} + 6\hat{j} + 2\hat{k}$$
$$\vec{c} = 3\hat{i} + 3\hat{j} - 6\hat{k}$$

(c) Find the general solution of the ordinary differential equation

$$y'' - 4y' + 4y = 0$$

given $y(0) = 3$, $y'(0) = 1$.

(d) Calculate the scale factors of cylindrical coordinate system.

(e) Ten randomly selected nails have the lengths (in inches)

0·80, 0·81, 0·81, 0·82, 0·81, 0·82, 0·80, 0·82, 0·81, 0·81

Calculate the mean and variance of this sample.

4. Using Gauss's divergence theorem, evaluate the following integral, where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular disks $z = 0$ and $z = b$ $(0 \le z \le b$ and $x^2 + y^2 \le a^2)$: 10

$$I = \iint_S (x^3 dy\,dz + x^2 y\,dz\,dx + x^2 z\,dx\,dy)$$

5. Verify the following identity : 10

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{V})$$

6. Obtain the expression for divergence in spherical polar coordinate system. 10

7. For a resistance-inductance circuit, Kirchhoff's law leads to

$$\frac{LdI(t)}{dt} + RI(t) = V(t)$$

for current $I(t)$ and voltage $V(t)$, where L, R are inductance and resistance. Obtain the general expression for $I(t)$. What is $I(t)$ for the special case $V(t) = V_0 = \text{constant}$? 10

8. Prove the following for Dirac-delta function : 3+3+4=10

(i) $\int_{-\infty}^{\infty} dx\, \delta(x) = 1$

(ii) $\delta(ax) = \frac{1}{a}\delta(x), \quad a > 0$

(iii) $\int_{-\infty}^{\infty} dx\, \delta(x-a) f(x) = f(a)$

9. (a) Explain the method of least square fitting giving all necessary details.

(b) What is meant by probability distribution function? Illustrate in detail about binomial, Gaussian and Poisson distribution functions. 5+5=10

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