3 (Sem–6) MAT M 2

2020

MATHEMATICS

(Major)

Paper : 6·2

(Numerical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

(a) If $\pi = \frac{22}{7}$ is approximated as 3·14, find the relative error and relative percentage error.

(b) Define 'absolute error'.

(c) Find the difference $\sqrt{2 \cdot 01} - \sqrt{2}$, correct to three significant figures.

Contd.

(d) If m and n are positive integers, then show that $\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$.

(e) Evaluate $\Delta^n\left(\frac{1}{x}\right)$, with 1 as the interval of differencing.

(f) Give the relationship between the operator Δ and the differential operator D.

(g) Write the general quadrature formula in numerical integration.

2. Answer the following questions : 2×4=8

(a) Find the number of significant figures in $x = 0{\cdot}3941$ whose absolute error is $0{\cdot}25 \times 10^{-2}$.

(b) Given $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$ and $u_5 = 8$, find $\Delta^5 u_0$.

(c) What is numerical differentiation ? Explain briefly its importance.

(d) Derive trapezoidal rule from Newton-Cotes quadrature formula.

3. Answer the following questions : 5×3=15

(a) Find the relative error for evaluation of $u = x_1 x_2$ with $x_1 = 4{\cdot}51$, $x_2 = 8{\cdot}32$ having absolute errors $\Delta x_1 = 0{\cdot}01$ in x_1 and $\Delta x_2 = 0{\cdot}01$ in x_2.

(b) Using the method of separation of symbols, prove the following :

$$(u_1 - u_0) - x(u_2 - u_1) + x^2(u_3 - u_2) - \ldots\ldots$$

$$= \frac{\Delta u_0}{1+x} - x\frac{\Delta^2 u_0}{(1+x)^2} + x^2 \frac{\Delta^3 u_0}{(1+x)^3} - \ldots.$$

Or

Find the function whose first difference is $9x^2 + 11x + 5$.

(c) A second degree polynomial passes through the points (1, −1), (2, −1), (3, 1) and (4, 5). Find the polynomial.

Or

Using Lagrange's interpolation formula, find the form of the function given by :

x :	3	2	1	−1
$f(x)$:	3	12	15	−21

4. Answer ***any one*** part :

(a) *(i)* Apply Stirling's formula to find a polynomial of degree 4 which takes the following tabular values :

x	:	1	2	3	4	5
$y=f(x)$	:	1	-1	1	-1	1

(ii) Using Newton's divided difference formula, construct the interpolating polynomial and hence compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=5$ using the following data :

x	:	0	2	3	4	7	9
y	:	4	26	58	112	466	922

5+5=10

(b) *(i)* Use Bessel's formula to find $y(0{\cdot}12)$ from the following data :

x	:	0	0·05	0·1	0·15	0·2	0·25
y	:	0	0·10017	0·20134	0·30452	0·41075	0·52110

(ii) Find the value of $\int_1^5 \log_{10} x \, dx$, taking 8 subintervals, by trapezoidal rule. 5+5=10

5. Answer ***any one*** part :

(a) (i) In a machine a slider moves along a fixed straight rod. Its distance x *cms* along the rod is given below for various values of time t seconds. Find the velocity and acceleration of the slider when $t = 0{\cdot}3$.

$t(sec)$:	0	0·1	0·2	0·3	0·4	0·5	0·6
$x(cm)$:	30·13	31·62	32·87	33·64	33·95	33·81	33·24

(ii) The velocity $v\,(km/min)$ of a car which starts from rest, is given at fixed intervals of time t (*min*) as follows :

t :	2	4	6	8	10	12	14	16	18	20
v :	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes. 5+5=10

(b) (i) Using Lagrange's formula and the following table, find $f'(3)$ and $f'(4)$:

x	: 1	2	4	8	10
$f(x)$	: 0	1	5	21	27

(ii) Find an approximate value of $log_e 7$ using Simpson's rule to the integral $\int_1^7 \frac{dx}{x}$.

5+5=10

6. Answer ***any one*** part :

(a) (i) Derive the rate of convergence of the Secant method.

(ii) Compute the root of $e^x - 3x = 0$, using bisection method, lying between 1·5 and 1·6, correct to two decimal places.

5+5=10

(b) (i) Using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$, which is nearer to $x=2$, correct to three decimal places.

(ii) Find an approximate root of the equation $x^3 + x - 1 = 0$ near $x = 1$, by the Regula-Falsi method, correct to two decimal places.

5+5=10
