3 (Sem–6) MAT M 5

2020

MATHEMATICS

(Major)

Paper : 6·5

(Graph and Combinatories)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

   (a) The value of $2P(n, n-2)$ is

   (i) $P(2n, n)$

   (ii) $P(n, n-2)$

   (iii) $P(n, n)$

   (iv) None of these.

(b) Find how many functions are there from X to Y where $X = \{1, 2, 3\}$, $Y = \{a, b, c\}$.

(c) The number of vertices of odd degree in a graph is —

(i) always even

(ii) always odd

(iii) can be even as well as odd

(iv) None of above.

(d) The number of vertex in a loop is :

(i) 0

(ii) 1

(iii) 2

(iv) 4

(e) Which of the following statements are true?

(i) Every cycle is a Hamiltonian graph.

(ii) Any graph obtained by adding edges to a Hamiltonian graph is also Hamiltonian.

(iii) A Hamiltonian graph always has a pendent vertex.

(iv) Trees are always Hamiltonian.

(f) Determine True ***or*** False of the following statement :

"$K_{3,3}$ is non-planar".

(g) Define Eulerian graph.

2. Answer the following questions : $2 \times 4 = 8$

(a) Prove that every graph is an intersection graph.

(b) Represent the graph $G(V, E)$ where the vertex set V and the edge set E are as follows :

$$V = \{1, 2, 3, 4\}$$

$$E = \{(x, y) : x + y \text{ is odd}\}$$

(c) A connected planar graph has nine vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there?

(d) Does there exist a tree T with 8 vertices such that, the sum of degree of vertices is 16? Justify your answer.

3. Answer the following questions : 5×3=15

(a) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3 and 7.

(b) There exists no simple graph corresponding to the following degree sequences 2, 2, 4, 4, 2. Justify the above statement.

Or

Show that a complete graph with n vertices consists of $\frac{n(n-1)}{2}$ edges.

(c) Prove that a connected graph is bipartite if and only if it contains no odd cycles.

Or

If a graph G is a tree then prove that every two vertices of G are joined by unique path.

4. Answer ***any one*** part :

(a) Prove that a connected graph G is Eulerian if and only if every vertex of G has even degree. 10

(b) (i) For a graph G, prove that

$K(G) \leq \lambda(G) \leq \delta(G)$

The symbols have their usual meaning. 6

(ii) Among all graphs with p vertices and q edges, prove that the maximum connectivity is 0 when

$q < p-1$ and $\left[\dfrac{2q}{p}\right]$ when $q \geq p-1$.

4

5. Answer ***any one*** part : 10

(a) If in a graph G has $n \geq 3$ vertices and every vertices has degree at least $\dfrac{n}{2}$, then G is Hamiltonian.

(b) Let G be a graph of n vertices. If the sum of the degrees of each pair of vertices in G is $n-1$ or larger, then prove that there exists a Hamiltonian path in G.

6. Answer ***any one*** part :

(a) (i) In how many ways can 21 identical books on English and 19 identical books on Hindi be placed in a row on a shelf, so that two books on Hindi may not be together?

4

(ii) Enumerate the number of non-negative integral to the inequality

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 19$$

6

(b) (i) How many outcomes are possible by casting a 6 faced die 10 times?

4

(ii) How many integers solution are there to the equation

$x_1 + x_2 + x_3 + x_4 = 13, \quad 0 \le x_i \le 5;$

$i = 1, 2, 3, 4$? 6