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# EFFECT OF POSITRON TEMPERATURE ON HIGH RELATIVISTIC ELECTRON-POSITRON-ION PLASMAS WITH NONTHERMAL ELECTRONS

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**Abstract**— In this model of relativistic electron-positron-ion plasma, only compressive solitons are found to exist. The amplitude becomes higher for smaller values of $\sigma = T_e / T_p$ (= electron to positron temperature ratio). The investigation further revealed that higher values of $v_0 / c$ ($v_0$ is the initial ion streaming and $c$ is the normalized velocity of light) gives higher amplitudes of compressive solitons.

**Keywords:** Relativistic effects, Positron temperature, Ion acoustic, Soliton.

## 1 INTRODUCTION

Waves in plasmas are an important phenomena interconnecting set of particles and fields which propagates periodically in a repeated fashion. A solitary wave is one propagates without any temporal evolution in shape and size and arises in many context like the elevation of the surface of water and the intensity of light in optical fibres. In the last few years, a considerable amount of interest has grown in the field of coherent nonlinear wave structures. Out of these nonlinear structures, ion acoustic solitons stands tall in modern plasma research. The first experimental observation of ion acoustic waves was made by Ikezi et al. [1]. Also many researchers [2, 3] have studied ion acoustic solitons using reductive perturbation method. A great deal of interest has risen recently in the study of nonlinear wave phenomena in electron-positron-ion plasma [4 – 9] which is mainly due to electron-positron-ion plasmas occur in many astrophysical environments such as active galactic nuclei [10], pulsar magnetosphere [11], polar regions of neutron stars [12], centre of our galaxy [13], the early universe [14,15] and solar atmosphere [16] and also produced in some laboratory environments [17 – 19]. Linear and nonlinear wave propagation in electron-positron and electron – positron – ion plasma has been studied using different models. For instance, Popel *et al.* [5] explored ion acoustic solitons in three component plasmas constituting electrons, positrons and singly charged ions giving a result that the presence of positron reduces ion acoustic amplitude. Moslem *et al*. [20] investigated the nonlinear two dimensional cylindrical acoustic excitations (solitons and double layers) in electron – positron – ion plasma having a composition of warm electrons and positive ions. However, most of these studies are focused on nonrelativistic plasmas, but when the particle velocities are comparable to the speed of light, relativistic effects may significantly modify the soliton behaviour [21 – 28]. Relativistic plasmas occur in a variety of situation like space plasmas [29], laser plasma interaction [30], plasma sheet boundary layer of earth's magnetosphere [31] and is also used in describing Van Allen Radiation belts [32]. Javidan and Saadatmand [33] have studied the effect of high relativistic ions and nonthermal electrons in electron-ion-positron plasma system. They have obtained the maximum amplitude of the solitary wave and its width as functions of plasma parameters. Sabarian et al. [34] have investigated the propagation of large amplitude ion-acoustic solitary waves in fully relativistic plasma consisting of cold ions and ultra-relativistic hot electrons and positrons using Sagdeev pseudo potential method. Shah et al [35] have studied the propagation of ion acoustic solitary waves in a plasma system comprising of relativistic ions, kappa distributed electrons, and positrons. They reported that the increase in the relativistic streaming factor and positron and electron kappa parameters cause the soliton amplitude to thrive. However, the soliton amplitude diminishes as the positron concentration is increased in the system. Saeed et al. [36] have investigatedabout the Nonlinear Korteweg–de Vries equation for soliton propagation in relativistic electron-positron-ion plasma with thermal ions.

In this paper, we have investigated effect of electron to positron temperature ratio in high relativistic electron-positron-ion plasma.

## 2 BASIC EQUATIONS AND DERIVATION OF KDV EQUATION

We consider collisionless, unmagnetized plasma consisting of positive ions, thermal positrons and nonthermal electrons. The fluid equations of motion, governing the collisionless plasma in one dimension are:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}\left(n_i v_i\right)=0 \tag{1}$$

$$\left(\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x}\right)\gamma v_i+\frac{\partial \phi}{\partial x}=0 \tag{2}$$

$$n_e=e^{\phi} \tag{3}$$

$$n_e = pe^{-\sigma\phi} \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2} = n_e - n_i - n_p \tag{5}$$

where $\gamma = \left(1 - \frac{v_i^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{v_i^2}{2c^2} + \frac{3v_i^4}{8c^4}$

where, $i$ ,$p$ and $e$ stand for positive ion, positron and electron respectively and $\sigma = T_e / T_p$ ( =electron to positron temperature ratio).

We normalize densities of the plasma species by the unperturbed densities $n_{e0}$, time t by the inverse of the characteristic ion plasma frequency i.e., $\omega_{pi}{}^{-1} = \left(\frac{m_i}{4\pi n_{e0} e^2}\right)^{\frac{1}{2}}$, distance x by the electron Debye length $\lambda_{De} = \left(\frac{T_e}{4\pi n_{e0} e^2}\right)^{\frac{1}{2}}$, velocities by the ion-acoustic speed $C_s = \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}$, and the potential $\phi$ by $\frac{T_e}{e}$.

Introducing the stretched variables

$$\xi = \varepsilon^{\frac{1}{2}}(x - Ut), \tau = \varepsilon^{\frac{3}{2}} t \tag{6}$$

where $\varepsilon$ is a small parameter which characterizes the strength of the nonlinearity and $U$ is the phase velocity. Dependent variables are expanded as follows:

$$n_i = 1 - p + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \ldots$$

$$n_e = 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \ldots$$

$$n_p = p + \varepsilon p_{p1} + \varepsilon^2 n_{p2} + \ldots \tag{7}$$

$$v_i = v_0 + \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \ldots$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \ldots$$

Using (6) and (7) in equations (1) - (5) and equating the coefficients of the lowest order perturbation in $\varepsilon$ with the use of the boundary conditions $n_{e1} - n_{i1} - n_{p1} = 0$, $v_{i1} = 0$ , $\phi_1 = 0$ at $|\eta| \to \infty$, we get

$$n_{i1} = \frac{1-p}{(U - v_0)^2 \beta}\phi_1, n_{e1} = \phi_1, n_{p1} = -\sigma\, p\phi_1, n_{e1} - n_{i1} - n_{p1} = 0 \tag{8}$$

From the last equation of (8), the expression for the phase velocity $U$ can be written as

$$1 - \frac{1-p}{(U - v_0)^2 \beta} + \sigma p = 0 \tag{9}$$

This gives

$$U = v_0 \pm \sqrt{\frac{1-p}{\beta(1+\sigma p)}}, \text{ where } p = \frac{n_{p0}}{n_{e0}} \tag{10}$$

From the second order equations in $\varepsilon$, KdV equation can be obtained as

$$\frac{\partial\phi_1}{\partial\tau} + p'\phi_1\frac{\partial\phi_1}{\partial\xi} + q'\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \tag{11}$$

where $p' = \frac{A}{B}$ and $q' = \frac{1}{B}$

with $A = \frac{3(1-p)}{(U - v_0)^4\beta^2} - \frac{2\beta_0(1-p)}{(U - v_0)^4\beta^3} - (1 - \sigma^2 p)$

and $B = \frac{2(1-p)}{(U - v_0)^3\beta}$

## 3 SOLITARY WAVE SOLUTION

The solution of the KdV equation (11) is obtained as

$$\phi_1 = \frac{3V}{p}\sec h^2\left(\frac{1}{2}\sqrt{\frac{V}{q}}\chi\right) \tag{12}$$

where $V$ represents the speed of the ion acoustic soliton with amplitude $\phi_0 = \frac{3V}{p}$ and width $\Delta = 2\sqrt{\frac{q}{V}}$. In deducing the solution (12), a new variable $\chi = \eta - V\tau$ is introduced and following boundary conditions are used $\phi_1 = \frac{\partial \phi_1}{\partial \eta} = \frac{\partial^2 \phi_1}{\partial \eta^2} = 0$ at $|\eta| \to \pm\infty$.

## 4 DISCUSSION

In this model of plasma, we have investigated the ion acoustic solitary waves in high relativistic electron-positron-ion plasma considering electron to positron temperature ratio $\sigma$, subject to the condition $0 < p \le 1$. The plasma parameters such as the relativistic factor $v_0 / c$, temperature ratio $\sigma$ and positron concentration $p$ are found to play a significant role in the formation of ion acoustic solitons. The amplitude of compressive KdV soliton [Fig. 1(a)] increases as $v_0 / c$ increases for fixed values of $V = 0.10$, $p = 0.50$ and for different values of $\sigma = 0.05, 0.10, 0.15$. From figure it is revealed that for higher temperature the amplitude becomes smaller. On the other hand, the width [Fig. 1(b)] of the compressive soliton decreases with $v_0 / c$ for the same set of parametric values. The amplitudes [Fig. 2(a)] of the compressive solitons showing linear decrease with $\sigma$ for fixed $V = 0.20$, $v_0 / c = 0.30$ and for different values of $p = 0.10(1), 0.20(2), 0.30(3)$. The corresponding widths [Fig. 2(b)] of the fast compressive solitons also decrease as figure 2(a).The amplitudes ($\phi_0$) [Fig. 3 (a)] of the compressive KdV solitons are seen to decrease sharply but linearly with $p$ for fixed $V = 0.20$, $v_0 / c = 0.30$ and for different values of $\sigma = 0.10, 0.30, 0.50$. But the corresponding widths $(\Delta)$ [Fig. 3(b)] decrease concavely with $p$. Figure 4 shows that the soliton height and width decreases as positron concentration $p$ increases. This situation indicates that the soliton energy decreases with an increasing $p$.

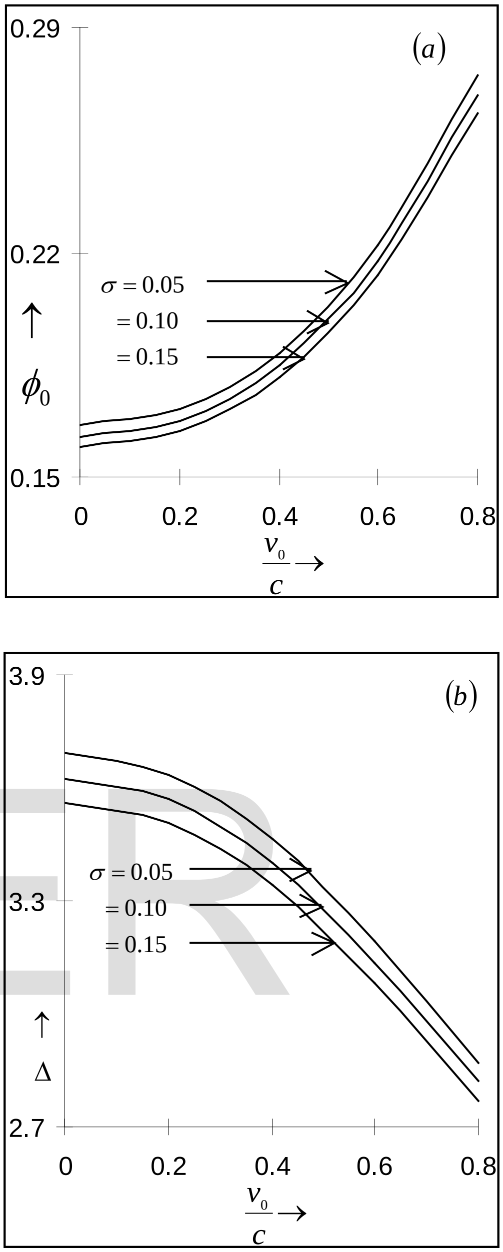

FIG.1. Amplitudes (a) and widths (b) of higher order relativistic compressive KdV solitons versus $\frac{v_0}{c}$ for fixed $V = 0.10$, $p = 0.50$ and for different values of $\sigma = 0.05, 0.10, 0.15$.

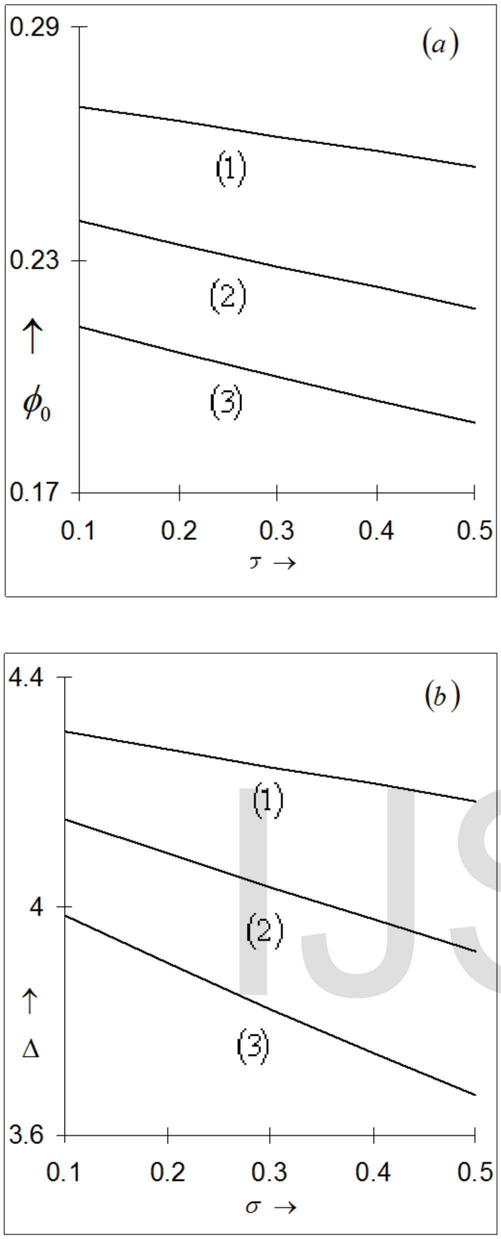

FIG.2. Amplitudes (a) and widths (b) of higher order relativistic compressive KdV solitons versus temperature ratio $\sigma$ for fixed $V = 0.20$, $\frac{v_0}{c} = 0.30$ and for different values of $p = 0.10(1),\ 0.20(2),\ 0.30(3)$.

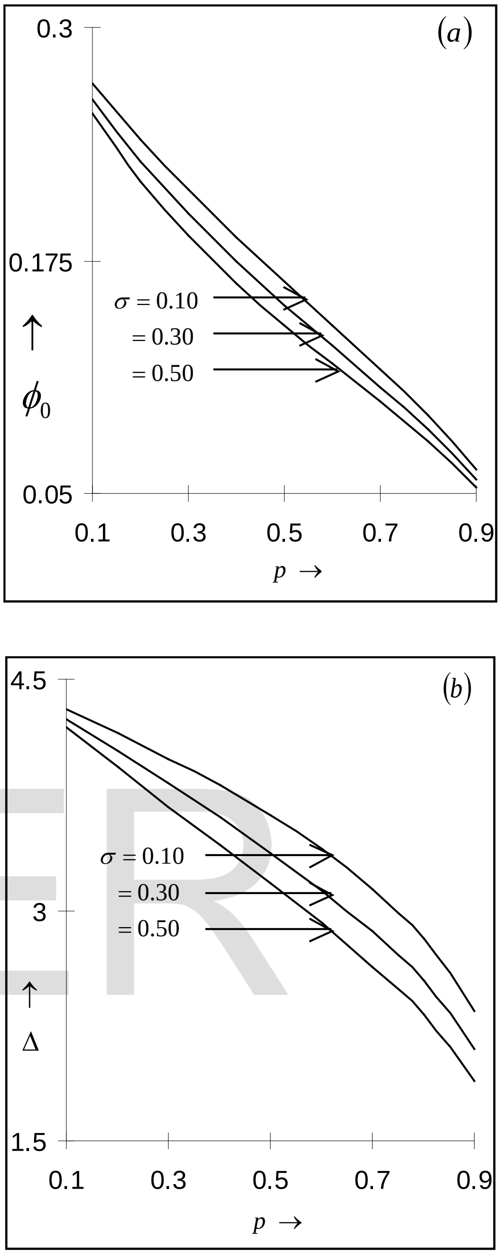

FIG.3. Amplitudes (a) and widths (b) of higher order relativistic compressive KdV solitons versus positron concentration $p$ for fixed $V = 0.20$, $\frac{v_0}{c} = 0.30$ and for different values of $\sigma = 0.10,\ 0.30,\ 0.50$.

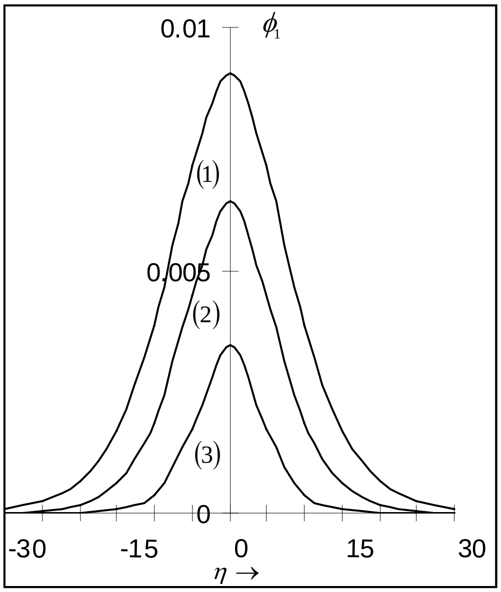

FIG.4. Plot of the amplitude of the compressive KdV solitons with $V = 0.0075$, $\sigma = 1$ and $v_0 / c = 0.30$ for different values of $p = 0.50(1), 0.70(2), 0.90(3)$.

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