3 (Sem–6) MAT M2

2022

MATHEMATICS

(Major)

Paper : 6·2

(Numerical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

   (a) If x is the true value of a quantity and x' is its approximate value, then what is the relative error ?

   (b) Define 'absolute error'.

   (c) Define 'truncation error'.

(d) Evaluate $\Delta^2 ae^{bx}$.

(e) Write down the Weddle's rule.

(f) Write the relationship between the operator Δ and the differential operator D.

(g) Evaluate

$$\Delta^{10}(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)$$

the interval of differencing being unity.

2. Answer the following questions : 2×4=8

(a) An approximate value of π is given by 3·1428571 and its true value is 3·1415926. Find the relative error.

(b) Show that

$$\Delta \log f(x) = \log\left\{1+\frac{\Delta f(x)}{f(x)}\right\}$$

(c) What is the degree of the approximating polynomial corresponding to Simpson's $\frac{3}{8}$th rule and Weddle's rule.

(d) Explain briefly the idea of numerical integration.

3. Answer the following questions :

(a) Round off the number 75462 to four significant digits and then calculate the absolute error and percentage error. 1+4=5

(b) Use the method of separation of symbols to prove the following identity :

$$u_1 x + u_2 x^2 + u_3 x^3 + \cdots = \frac{x}{1-x} u_1 + \frac{x^2}{(1-x)^2} \Delta u_1 + \frac{x^3}{(1-x)^3} \Delta^2 u_1 + \cdots$$

where $u_{x+h} = E^h u_x$. 5

(c) Derive Lagrange's interpolation formula and mention *two* advantages of it. 3+2=5

Or

Use Newton's forward interpolation formula

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 y_0 + \cdots$$

where $u = \frac{x - x_0}{h}$ to establish the formula

$$\left(\frac{d^2y}{dx^2}\right)_{x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0 + \cdots\right] \quad 5$$

4. Answer ***either*** *(a)* ***or*** *(b)* :

(a) *(i)* What do you mean by 'divided difference'? Show that the divided differences are independent of the order of the arguments. 1+4=5

(ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's $\frac{3}{8}$th rule. 5

(b) (i) What do you mean by 'numerical differentiation'? Explain briefly with the help of a suitable example. 2+3=5

(ii) Using Gauss's forward formula, find y for $x = 3{\cdot}75$, given that

x :	2·5	3·0	3·5	4·0	4·5	5·0
y :	24·145	22·043	20·225	18·644	17·262	16·047

5

5. Answer **either** (a) **or** (b) :

(a) (i) Find $f'(6)$ from the following table : 5

x :	0	1	3	4	5	7	9
$f(x)$:	150	108	0	−54	−100	−144	−84

(ii) The velocity v (km/min) of a car which starts from rest, is given at fixed intervals of time t (min) as follows :

t :	2	4	6	8	10	12	14	16	18	20
v :	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes. 5

(b) (i) Compute the integral $\int_5^{12} \frac{dx}{x}$ by applying Gauss's quadrature formula. Also find the error.

4+1=5

(ii) Derive Newton-Cotes quadrature formula. How can the trapezoidal rule be deduced from it?

3+2=5

6. Answer ***either*** *(a)* ***or*** *(b)* :

(a) (i) Derive the Newton-Raphson formula and show that the rate of convergence is quadratic.

3+2=5

(ii) Find an approximate root of the equation $x^3 - 4x - 9 = 0$ using the bisection method four times. 5

(b) (i) Explain the bisection method with suitable diagram. 5

(ii) Find the root of the equation $xe^x = \cos x$ using secant method, correct to four decimal places (take the initial approximations as 0 and 1). 5
