Total number of printed pages–7

14 (MAT–1) 1026 (N)

2021

(Held in 2022)

MATHEMATICS

Paper : MAT–1026

(New Course)

(Mathematical Methods)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any three*** parts : 5×3=15

(*a*) Transform the boundary value problem

$$\frac{d^2\phi}{dx^2} + x\phi = 1\,;\ \phi(0)=0,\ \phi(1)=1$$

into integral equation.

(b) Find the resolvent kernel of the integral equation

$$\phi(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 x t \phi(t)\, dt$$

and hence solve it.

(c) Find the eigenvalues and eigenfunctions of the integral equation

$$\phi(x) = \lambda \int_0^\pi \cos(x+t)\, \phi(t)\, .dt$$

(d) Solve the non-homogeneous integral equation

$$\phi(x) = \sec^2 x + \lambda \int_0^1 \phi(t)\, dt$$

2. Answer ***any two*** parts : 5×2=10

(a) Using the method of successive approximation, solve the integral equation

$$\phi(x) = 1 + \int_0^x \phi(t)\, dt$$

when the initial approximation is $\Phi_0(x) = 0$.

(b) Show that the function $\phi(x) = 1 - x$ is a solution of the integral equation

$$\int_0^x e^{x-t}\phi(t)\,dt = x$$

(c) Solve :

$$\phi(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t}\phi(t)\,dt$$

3. Answer ***any three*** questions : 5×3=15

(a) Show that

$$f(x) = \frac{1}{\pi}\int_{\alpha=0}^{\infty}\int_{u=-\infty}^{\infty} f(u)\cos\{\alpha(x-u)\}\,du\,d\alpha$$

and

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\alpha u}\,d\alpha \int_{-\infty}^{\infty} f(u)e^{-i\alpha u}\,du \text{ are}$$

equivalent.

(b) Solve for $y(x)$, where the integral equation is

$$\int_{-\infty}^{\infty} \frac{y(u)du}{(x-u)^2+x^2} = \frac{1}{x^2+b^2} \; ; \quad 0<a<b$$

(c) Find the Fourier cosine transform of e^{-x^2}.

(d) Solve $\partial U/\partial t = 2\left(\partial^2 U/\partial x^2\right)$ if $U(0, t)=0$, $U(x, 0)=e^x$, $x>0$, $U(x, t)$ is bounded where $x>0, t>0$.

4. (a) Show that the function e^{t^2} is not of exponential order. 3

(b) If $L\{f(t)\}=F(s)$ and $L\{g(t)\}=G(s)$, then prove that $L(f*g)=F(s)G(s)$. 5

Or

Obtain $L^{-1}\left\{\frac{s}{(s-1)^2(s-2)}\right\}$ by complex inversion formula.

(c) If $L\{f(t)\} = F(s)$ and given that $L\{f'(t)\} = sF(s) - f(0)$. Then find $L\{\cos at\}$ if $f(t) = \sin at$. 2

(d) If $L\{f(t)\} = F(s)$, prove that

$$L\left\{\int_0^t f(u)\,du\right\} = \frac{F(s)}{s}.$$ 5

Or

Solve by Laplace transformation method

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t},$$

$y(0) = 0,\ y'(0) = 0.$

5. (a) Test for extremum for the functional

$$\int_1^e \left(xe^y - ye^x\right)dx,\ y(1)=1,\ y(e)=e$$ 3

(b) Extremize $\int_0^1 \left(x \sin y' + \cos y\right)dx$ with

$y(0) = 0,\ y(1) = \frac{\pi}{2}$ 2

6. Answer ***any two*** parts : 5×2=10

(a) Find the extremal of the functional

$$\int_0^{\pi/2} \left(y'^2 - y^2 + 2xy\right) dx$$

with $y(0) = 0$ and $y(\pi/2) = 0$.

(b) Find the curve on which a particle will slide from one point to another point in the shortest time under the action of gravity. (Friction and resistance of media are ignored)

(c) Show that the geodesics on right circular cylinder is a circular helix.

7. Answer ***any two*** parts : 5×2=10

(a) Extremize $\int_0^{\pi/2} \left(y'^2 + z'^2 + 2yz\right) dx$ with

$y(0) = 0, \; y(\pi/2) = 1, \; z(0) = 0, \; z(\pi/2) = 1$

(b) Find the extremal of functional

$$I=\int_0^{\pi}\left(y'^2-y^2\right)dx$$

under conditions $y(0)=0,\ y(\pi)=1$ and subject to constraint

$$\int_0^{\pi} y\,dx=1.$$

(c) Find the solid of maximum volume formed by the revolution of a given surface.