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14 (MAT–1) 1056 (N)

2021

(Held in 2022)

MATHEMATICS

Paper : MAT–1056

(New Course)

(Numerical Analysis)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any four*** questions : 5×4=20

(a) Solve the system of equations using the Gauss elimination method with partial pivoting :

$$2x_1 + x_2 + x_3 - 2x_4 = -10$$

$$4x_1 + 2x_3 + x_4 = 8$$

$$3x_1 + 2x_2 + 2x_3 = 7$$

$$x_1 + 3x_2 + 2x_3 - x_4 = -5$$

(b) Solve the following system of equations by LU decomposition method :

$$x_1 + x_2 - x_3 = 2$$

$$2x_1 + 3x_2 + 5x_3 = -3$$

$$3x_1 + 2x_2 - 3x_3 = 6$$

(c) Solve the following system of equations using the Jacobi iteration method :

$$4x_1 + x_2 + x_3 = 2$$

$$x_1 + 5x_2 + 2x_3 = -6$$

$$x_1 + 2x_2 + 3x_3 = -4$$

Take the initial approximation as $x^{(0)} = [0{\cdot}5 \;\; -0{\cdot}5 \;\; -0{\cdot}5]^T$ and perform three iterations.

(d) State and prove Gerschgorin's theorem.

(e) Let A be a square matrix. Prove that the infinite series $I + A + A^2 + \dots$ converges if and only if $\lim_{m \to \infty} A^m = 0$. Moreover, show that (in case of convergence) the series converges to $(I - A)^{-1}$.

2. Answer ***any two*** questions : 5×2=10

(a) Discuss Aitkin's Δ^2 method by using general iteration method.

(b) Perform three iterations of the Newton-Raphson method to solve the system of equations :

$$x^2 + xy + y^2 = 7$$

$$x^3 + y^3 = 9$$

Take the initial approximation as $x_0 = 1{\cdot}5,\ y_0 = 0{\cdot}5$.

(c) Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial

$$p_3(x) = x^3 + x^2 - x + 2 = 0.$$

Use the initial approximation $p_0 = -0{\cdot}9$ and $q_0 = 0{\cdot}9$.

3. *(a)* Given the following values of $f(x)$ and $f'(x)$ —

x	$f(x)$	$f'(x)$
−1	1	−5
0	1	1
1	3	7

estimate the values of $f(-0{\cdot}5)$ and $f(0{\cdot}5)$ using Hermite interpolation. 10

Or

Obtain the piecewise cubic interpolating polynomial for the function $f(x)$ defined by the data :

x :	-5	−4	−2	0	1	3	4
$f(x)$:	275	−94	−334	−350	−349	−269	−94

Hence find an approximate value of $f(-3{\cdot}0)$. 9+1=10

(b) Compute the value of x for $y = f(x) = 0{\cdot}6001$ from the following table using successive approximation method. 5

x :	2·0	2·1	2·2	2·3	2·4	2·5	2·6
y :	0·6020	0·6128	0·6232	0·6335	0·6434	0·6532	0·6628

4. *(a)* Derive Gauss-Legendre three-point integration method and write the error term. Using this formula, evaluate the following integral :

$$\int_1^2 \frac{2x}{1+x^4} dx$$

6+1+3=10

Or

Evaluate $\int_0^1 \left(1+\frac{\sin x}{x}\right) dx$ using composite trapezoidal rule with 2,3,5 nodes and Romberg integration. Assume $f(0)$ is taken as the limiting value. 10

(b) Using Euler-Maclaurin's summation formula, evaluate the following :

$$\frac{1}{(201)^2}+\frac{1}{(203)^2}+\ldots+\frac{1}{(299)^2}.$$

5

5. Answer ***any two*** parts : 10×2=20

(a) Consider the initial value problem :

$$y' = x(y+x) - 2, \ y(0) = 2.$$

Use Euler's method with step-sizes $h = 0 \cdot 3$, $0 \cdot 2$ and $0 \cdot 15$ to compute approximation to $y(0 \cdot 6)$.

(b) Given $y' = \frac{1}{2}(1+x^2)y^2, \ y(0) = 1.$

Evaluate $y(0 \cdot 4)$ by modified Euler's method with step length $h = 0 \cdot 1$.

(c) Use the classical R–K method of fourth order to find numerical solution at $x = 0 \cdot 8$ for $y' = \sqrt{x+y}, \ y(0 \cdot 4) = 0 \cdot 41$. Assume the step length $h = 0 \cdot 2$.