2019

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods—II)

(*Marks* : 35)

1. Answer the following questions : 1×3=3

(a) State the condition for which any vector $\vec{A}$ is conservative.

(b) Define unit vector in curvilinear coordinate system.

(c) Evaluate $\overline{|0}$.

2. If $\vec{F} = \cos y\hat{i} - x\sin y\hat{j} - \cos z\hat{k}$, show that the field is conservative. 2

3. Answer any *two* of the following questions : 5×2=10

(a) *(i)* Find the work done in moving an object in the field

$$\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$$

from (1, −2, 1) to (3, 1, 4). 3

(ii) Show that the area bounded by a simple closed curve C is given by

$$\frac{1}{2}\oint_C (x\,dy - y\,dx)$$

2

(b) *(i)* Express $\vec{\nabla} \times \vec{A}$ in spherical coordinates. 2

(ii) Write down the transformation equations of orthogonal cylindrical and spherical coordinate systems from Cartesian coordinate and provide the limit of different parameters. 3

(c) *(i)* Evaluate

$$\int_0^\infty \frac{x^a}{a^x}\,dx$$

using gamma function. 2

(ii) Show that

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$

for any non-zero value of a. 3

4. Answer any *two* of the following questions : 10×2=20

(a) *(i)* If $\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$, evaluate

$$\iiint_V \vec{F}\, dV$$

where V is the region bounded by the surfaces $x = 0$, $y = 0$; $x = 2$, $y = 4$, $z = x^2$, $z = 2$. 5

(ii) A vector field $\vec{F}$ is given by

$$\vec{F} = \sin y\hat{i} + x(1 + \cos y)\hat{j}$$

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the circular path given by $x^2 + y^2 = a^2$. 5

(b) *(i)* Find the scalar potential of

$$\vec{A} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$$ 5

(ii) If S be a closed surface and $\vec{r}$ be the position vector of a point (x, y, z) with respect to an origin O, then prove that

$$\iint_S \frac{\vec{n} \cdot \vec{r}}{r^3} dS = 4\pi$$

when O lies inside of S. 5

(c) (i) Express $\vec{\nabla}\phi$ and $\vec{\nabla}\cdot\vec{A}$ in cylindrical coordinates. 2+3=5

(ii) Prove that a spherical coordinate system is orthogonal. 5

GROUP—B

(Properties of Matter)

(*Marks* : 25)

5. Answer the following questions : 1×4=4

(a) For a given material, the value of Young's modulus (Y) is 2·4 times of its shear modulus. Find the value of Poisson's ratio.

(b) Define beam and cantilever.

(c) What will be the surface tension of a liquid at critical temperature?

(d) How does the velocity of propagation of waves passing over the surface of a liquid depend on gravity?

6. Answer the following questions : 2×3=6

(a) If you ride your bicycle directly behind a large truck, you will find that you don't have to pedal hard to keep moving forward. Why?

(b) Show that for an elastic material, the value of Poisson's ratio varies from -1 to $0{\cdot}5$.

(c) Draw the resultant force of surface tension of a plane, concave and convex surface.

7. Answer any *one* of the following questions :

(a) (i) Show that shear (θ) is equal to half a tensile and half a compressive strain at right angles to each other. 3

(ii) Find the work done in stretching a wire of 1 sq. mm cross-section, Young's modulus 2×10^{11} N/m^2 and 2 m long through $0{\cdot}1$ mm. 2

(b) (i) Why is correction of Poiseuille's equation necessary? 3

(ii) A rigid rod $1{\cdot}5$ m long is fixed horizontally at one end and loaded at the other by a mass of $0{\cdot}1$ kg. Calculate the depression of a point distant $1{\cdot}2$ m from the fixed end. Diameter of the rod is 2 cm. Young's modulus for the material of the rod is $1{\cdot}01\times10^{11}$ N/m^2. 2

8. Answer *either (a) or (b)* :

(a) *(i)* Establish the relation $Y = 3K(1-2\sigma)$, where Y is Young's modulus, K is bulk modulus and σ is Poisson's ratio.

The stress to strain ratio remains constant for a small deformation. What happens to this ratio, if deformation is made very large and why? 5+2=7

(ii) Determine the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm. For glass, bulk modulus = 37×10^9 Nm^{-2}. 3

(b) *(i)* Using the relation for excess pressure acting on a curved surface

$$P = 2T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

evaluate excess pressure for spherical liquid drop, air bubbles in liquid drop, soap bubble and cylindrical film.

Explain why the free surface of a liquid behaves like a stretched membrane. 4+3=7

(ii) A liquid drop of radius R breaks into 64 tiny drops. Find the resulting change in energy in terms of surface tension T. 3

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